FREE CASH FLOW MODELS, TERMINAL VALUES AND THE TIMING OF ASSET REPLACEMENTS

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Abstract

This paper analyses the issue of the timing of expenditures in replacing fixed assets within the context of valuing firms using the free cash flow approach. Standard practice amongst both practitioners and academics is to assume a smooth pattern in these expenditures past some future point, and such a pattern is improbable. This paper shows how to correctly address this issue, that failure to do so may induce a valuation error in excess of 50%, and presents an example from a company in which this appears to be the case. In conducting the analysis, it is important to know the current replacement costs of the firm's existing assets, their economic lives and their residual lives. The average values for the last two pieces of information can be deduced from either historic cost or replacement cost valuation of the firm's assets, and average values are sufficient for this purpose. By contrast, the first of these three pieces of information requires replacement cost valuation of the firm's assets, and therefore Financial Statements based upon replacement cost asset valuations are superior to those based upon historic cost valuations.

1. Introduction

A widely used method for valuing companies involves discounting of "free cash flows" up to some horizon along with a "terminal value". The latter is generally obtained by applying some multiple to expected earnings at that point, or invoking a discounted cash flow approach with simple assumptions concerning the expected growth in cash flows from that point (Bodie et al, 1996; Brealey and Myers, 2003; Copeland et al, 2005). Implicit in such processes is the assumption that expenditures for replacing fixed assets are expected to grow at a constant rate beyond the horizon point. This process is subject to at least two major concerns. The first is the failure to recognise that the timing of asset replacements is stochastic, and this has been addressed (Mauer and Ott, 1995; Dobbs, 2004). The second issue is the failure to explicitly consider the age profile of a firm's assets at the horizon point; consideration of it may rebut the assumption that replacements expenditures are expected to grow at a constant rate even when the timing of replacements is deterministic. Furthermore, the valuation error resulting from this second issue could be very substantial.

This paper seeks to investigate this second issue. To separate this issue from that of stochastic timing of asset replacements, we assume that the timing of asset replacements is deterministic. Section 2 develops the relevant theory. Section 3 considers a stylised example, and finds that the firm's value could be overestimated by more than 100% through failure to explicitly consider this issue. Section 4 then considers a real-world example, and finds that similarly large overestimates are possible. Section 5 concludes.

2. Theory

Let F_t^u denote unlevered cash flow from operations net of replacement and new investment for year t, k denote the weighted average cost of capital and V_T the value of the firm in T years. The current value of the firm is then as follows.

$$V_0 = \sum_{t=1}^{T} \frac{E(F_t^u)}{(1+k)^t} + \frac{E(V_T)}{(1+k)^T} \qquad (1)$$

The terminal value V_T is generally obtained by applying some multiple to earnings at that point, or invoking a discounted cash flow (DCF) approach with simple assumptions concerning the expected growth in free cash flows from that point. For example, Bodie et al (1996, p. 543) adopt the DCF approach with a constant growth rate of 6% and T = 1, Brealey and Myers (2003) adopt the DCF approach with a constant growth rate of 6% and T = 6, and Copeland et al (2005, Ch. 14) adopt the DCF approach with a constant growth rate of 5% and T = 10. We also adopt the DCF approach, along with the conservative assumption of no new investment after the terminal year (T) because the level of new investment is not relevant to the issue analysed here [1]. Consistent with this assumption, it would be sensible to assume an expected growth rate in free cash flows from year T+1 equal to some expected inflation rate, denoted i. To overcome the problem that the free cash flow in year T+1 incorporates the replacement investment in that year, and the level of the latter may be untypical, we substitute accounting depreciation in year T+1 (denoted AD_{T+1}) for replacement investment, i.e., defining W_{T+1} as the unlevered cash flow in year T+1 from operations *before* any deduction for replacement investment, and defining

$$X_{T+1} = W_{T+1} - AD_{T+1} \tag{2}$$

the expected value of the firm in year T is then as follows.

$$E(V_T) = \frac{E(X_{T+1})}{1+k} + \frac{E(X_{T+1})(1+i)}{(1+k)^2} + ... = \frac{E(X_{T+1})}{k-i} \tag{3}$$

Substitution of equation (3) into equation (1) yields the following.

$$V_0 = \sum_{t=1}^{T} \frac{E(F_t^u)}{(1+k)^t} + \frac{\left[\dfrac{E(X_{T+1})}{k-i}\right]}{(1+k)^T} \tag{4}$$

This approach ignores the actual timing of replacement investment, and we now seek to properly account for that. To begin, suppose that the firm has one fixed asset, for

[1] The same result arises if one assumes that all new investment after year T has zero NPV.

which the tax saving arising from tax depreciation in year T+1 will be TS_{T+1}, the asset is expected to be replaced M years subsequent to year T at a cost (net of any salvage value) of C_{M+T}, and it has an economic life of N years. The cash flow from operations W_{T+1} includes the tax saving TS_{T+1}. So, define Z_{T+1} as W_{T+1} stripped of this tax savings, i.e.,

$$Z_{T+1} = W_{T+1} - TS_{T+1} \tag{5}$$

The correct specification for $E(V_T)$ is then as follows.

$$E(V_T) = \frac{E(Z_{T+1})}{k-i} + \sum_{t=1}^{M} \frac{TS_{T+t}}{(1+k)^t} - \frac{E(C_{M+T})}{(1+k)^M} + \sum_{t=M+1}^{M+N} \frac{E(TS_{T+t})}{(1+k)^t} - .....$$

The first term on the right hand side here is the present value of all cash flows after year T except those relating to the replacement of this asset and the tax savings arising from tax depreciation. The second term is the present value of the tax savings arising from tax depreciation on the asset from year T+1 until the end of its economic life. The third term is the present value of the net replacement cost of the asset in N years. The fourth term is the present value of the tax savings arising from tax depreciation on this replacement asset, over its economic life. The last two terms have counterparts at intervals of N years out to infinity. The depreciation tax saving arising in year t of this asset's life will be some proportion of the cost of the asset, and is denoted P_t. So, the last equation can be written as follows.

$$E(V_T) = \frac{E(Z_{T+1})}{k-i} + \sum_{t=1}^{M} \frac{TS_{T+t}}{(1+k)^t} - \frac{E(C_{M+T})}{(1+k)^M}\left[1 - \sum_{t=1}^{N} \frac{P_t}{(1+k)^t}\right] - \frac{E(C_{M+T+N})}{(1+k)^{M+N}}\left[1 - \sum_{t=1}^{N} \frac{P_t}{(1+k)^t}\right] - ...$$

Implicit in the application of expected growth rate i to all components of X_{T+1} is the application of this rate i to the net replacement costs of the asset. We follow this standard practice. So, letting the current replacement cost of the asset (new) net of salvage value be denoted C_0, the last equation can be written as follows.

$$E(V_T) = \frac{E(Z_{T+1})}{k-i} + \sum_{t=1}^{M} \frac{TS_{T+t}}{(1+k)^t} - \frac{C_0(1+i)^{M+T}}{(1+k)^M}\left[1 - \sum_{t=1}^{N} \frac{P_t}{(1+k)^t}\right] - \frac{C_0(1+i)^{M+T+N}}{(1+k)^{M+N}}\left[1 - \sum_{t=1}^{N} \frac{P_t}{(1+k)^t}\right] - ...$$

All terms beyond the first two on the right hand side are now a geometric progression out to infinity. So, the last equation can be written as follows.

$$E(V_T) = \frac{E(Z_{T+1})}{k-i} + \sum_{t=1}^{M} \frac{TS_{T+t}}{(1+k)^t} - \frac{\dfrac{C_0(1+i)^{M+T}}{(1+k)^M}\left[1 - \sum_{t=1}^{N} \dfrac{P_t}{(1+k)^t}\right]}{1 - \dfrac{(1+i)^N}{(1+k)^N}} \qquad (6)$$

Substitution of equation (6) into equation (1) yields the following current value for the firm.

$$V_0 = \sum_{t=1}^{T} \frac{E(F_t^u)}{(1+k)^t} + \frac{\left[\dfrac{E(Z_{T+1})}{k-i} + \sum_{t=1}^{M} \dfrac{TS_{T+t}}{(1+k)^t} - \dfrac{\dfrac{C_0(1+i)^{M+T}}{(1+k)^M}\left[1 - \sum_{t=1}^{N} \dfrac{P_t}{(1+k)^t}\right]}{1 - \dfrac{(1+i)^N}{(1+k)^N}}\right]}{(1+k)^T} \qquad (7)$$

This formula assumes the presence of only one fixed asset. However, there will in general be multiple assets (*j* = 1, 2…*S*). For asset *j*, let the current replacement cost net of salvage value be denoted C_{j0}, the term to first replacement after time *T* be denoted M_j, the economic life be denoted N_j, and the tax savings from tax depreciation (in year *t* of the asset's life) as a proportion of the asset cost be denoted P_{jt}. The current value of the firm is then as follows.

$$V_0 = \sum_{t=1}^{T} \frac{E(F_t^u)}{(1+k)^t} + \frac{\dfrac{E(Z_{T+1})}{k-i} + \sum_{j=1}^{S}\left[\sum_{t=1}^{M_j} \dfrac{TS_{jT+t}}{(1+k)^t} - \dfrac{\dfrac{C_{j0}(1+i)^{M_j+T}}{(1+k)^{M_j}}\left[1 - \sum_{t=1}^{N_j} \dfrac{P_{jt}}{(1+k)^t}\right]}{1 - \dfrac{(1+i)^{N_j}}{(1+k)^{N_j}}}\right]}{(1+k)^T} \qquad (8)$$

The first term here is the present value of the free cash flows for the next *T* years. The second term is the present value of the subsequent operating cash flows prior to any deductions for replacement investment and their associated tax effects. The third term is the present value of the depreciation tax savings from all assets in place at time *T* until their first replacement. The last term is the present value of all replacement expenditures on these assets along with all subsequent depreciation tax savings.

In the face of many minor assets, a simplification of equation (7) would be to restrict the third and fourth terms to the major fixed assets (j = 1, 2…Q), and therefore leave the allowance for depreciation net of the tax effect on the remaining assets within the second term. So, the current value of the firm would be estimated as follows.

$$V_0 = \sum_{t=1}^{T} \frac{E(F_t^u)}{(1+k)^t} + \frac{\dfrac{E(Y_{T+1})}{k-i} + \sum_{j=1}^{Q} \left[\sum_{t=1}^{M_j} \dfrac{TS_{jT+t}}{(1+k)^t} - \dfrac{\dfrac{C_{j0}(1+i)^{M_j+T}}{(1+k)^{M_j}} \left[1 - \sum_{t=1}^{N_j} \dfrac{P_{jt}}{(1+k)^t} \right]}{1 - \dfrac{(1+i)^{N_j}}{(1+k)^{N_j}}} \right]}{(1+k)^T} \qquad (9)$$

where Y_{T+1} is the unlevered cash flow from operations in year T+1 less the accounting depreciation for the minor assets less the depreciation tax savings for the major assets. These valuation formulas in equations (8) and (9) can now be compared with equation (4). Although equations (8) and (9) seem formidable relative to equation (4), all of the additional parameters are observable by at least corporate insiders.

3. A Stylised Example

To explore the extent of variations between equations (4) and (8), consider the following stylised example. A firm currently has unlevered cash flow from operations of W_0 = \$100m, which is expected to grow at 2% per annum indefinitely. The firm has one fixed asset, which requires replacement every 20 years, and no further investment is anticipated. The current cost of replacing this asset (new) is C_0 = \$1100m, this cost has grown at 2% per annum since the last replacement, and is expected to grow at this rate indefinitely. Also, the asset's salvage value will be zero, the firm's weighted average cost of capital is k = 10%, the corporate tax rate is 33%, and straight line depreciation is used by both the firm and the tax authorities. We set T = 5 and examine various possible values for M.

We start with M = 1, i.e., the first replacement of the asset will arise one year after the horizon date of T = 5. So, the last replacement was 14 years ago. Given C_0 = $1100m and a historic inflation rate here of 2% per annum, the historical cost of the asset must have been $833.7m, which in turn implies AD_6 = $41.7m and TS_6 = $13.8m. Following equations (2), (3) and (4)

$$E(X_6) = \$100m(1.02)^6 - \$41.7m = \$70.9m$$

$$E(V_5) = \frac{\$70.9m}{.10 - .02} = \$886.2m$$

$$V_0 = \frac{\$102m}{1.10} + \frac{\$104m}{(1.10)^2} + \frac{\$106.1m}{(1.10)^3} + \frac{\$108.2m}{(1.10)^4} + \frac{\$110.4m + \$886.2m}{(1.10)^5} = \$951m$$

By contrast, invoking equations (5), (6) and (7)

$$E(Z_6) = \$100m(1.02)^6 - \$13.8m = \$98.8m$$

$$\sum_{t=1}^{20} \frac{P_t}{(1+k)^t} = .33\left(\frac{1}{20}\right)\left[\frac{1}{1.10} + ...\frac{1}{(1.10)^{20}}\right] = .14$$

$$E(V_5) = \frac{\$98.8m}{.10 - .02} + \frac{\$13.8m}{1.10} - \frac{\frac{\$1100m(1.02)^6}{(1.10)}[1 - .14]}{1 - \frac{(1.02)^{20}}{(1.10)^{20}}} = \$5m$$

$$V_0 = \frac{\$102m}{1.10} + \frac{\$104m}{(1.10)^2} + \frac{\$106.1m}{(1.10)^3} + \frac{\$108.2m}{(1.10)^4} + \frac{\$110.4m + \$5m}{(1.10)^5} = \$404m$$

So, equation (4) overestimates the value of the firm by 135%, and is the largest overstatement obtainable across the range of admissible values for M.

At the other extreme, M = 15 years, i.e., the asset has just been replaced for C_0 = \$1100m, leading to AD_6 = \$55m and TS_6 = \$18.2m. Following equations (2), (3) and (4)

$$E(X_6) = \$100m(1.02)^6 - \$55m = \$57.6m$$

$$E(V_5) = \frac{\$57.6m}{.10-.02} = \$720m$$

$$V_0 = \frac{\$102m}{1.10} + \frac{\$104m}{(1.10)^2} + \frac{\$106.1m}{(1.10)^3} + \frac{\$108.2m}{(1.10)^4} + \frac{\$110.4m + \$720m}{(1.10)^5} = \$848m$$

By contrast, invoking equations (5), (6) and (7)

$$E(Z_6) = \$100m(1.02)^6 - \$18.2m = \$94.4m$$

$$\sum_{t=1}^{15} \frac{TS_{5+t}}{(1+k)^t} = \$18.2\left[\frac{1}{1.10} + ...\frac{1}{(1.10)^{15}}\right] = \$138.4m$$

$$E(V_5) = \frac{\$94.4m}{.10-.02} + \$138.4m - \frac{\frac{\$1100m(1.02)^{20}}{(1.10)^{15}}\left[1-.14\right]}{1-\frac{(1.02)^{20}}{(1.10)^{20}}} = \$886.5m$$

$$V_0 = \frac{\$102m}{1.10} + \frac{\$104m}{(1.10)^2} + \frac{\$106.1m}{(1.10)^3} + \frac{\$108.2m}{(1.10)^4} + \frac{\$110.4m + \$886.5m}{(1.10)^5} = \$951m$$

Equation (4) now underestimates the value of the firm by 11%. So, across the range of values for M from 1…15 years, equation (4) could underestimate the value of the firm by as much as 11% and overestimate it by as much as 135%. These errors arise solely from ignoring the correct timing of asset replacements.

4. A Real World Example

Have illustrated the potential for error in equation (4) from a stylised example, we now consider a more realistic case. This involves Telecom New Zealand, which is the largest company listed on the New Zealand Stock Exchange. The most recently available Financial Statements are for the year ended 30.6.2005, and reveal cash flows from operations prior to any deduction for replacement or new investment (W_0) of approximately 1900m.[2] Purchases of fixed assets averaged $820m per year over the previous five years, with figures ranging from $600m to $1500m[3]. These expenditures on fixed assets include replacement investment, but they may include some new investment as well. Furthermore, new investment can take the form of purchasing stakes in other companies, and Telecom's net investments of this type in the last five years have averaged $240m per year, but with considerable variation. So, total investment has averaged $1060m per year over the previous five years, and with considerable variation. By contrast, the figures for depreciation have been much more stable over that period, with a range from about $650m to $750m and a current level of $700m.

All of this information is historic, and gives only limited insight into its anticipated expenditures for replacement and new investment over the next $T = 5$ years. However, our concern lies in comparing the outcomes from equations (4) and (8), and projected expenditures for replacement investment over the next five years are common to both approaches. So, in the interests of simplification, we assume replacement expenditures of R_t = $650m per year[4]. For the same reason, we assume no new investment at any future point. Consistent with assuming no new investment, we project W_t to grow at some expected inflation rate i from the current figure of W_0 = $1900m. We also set $i = .02$, consistent with recent long-run forecasts of inflation in New Zealand consumer prices (The Treasury, 2005). The expected free cash flows

2 The Statement of Cash of Cash Flows reports net cash flows to equityholders from operating activities of $1703m. Adding back the interest payments of $318m, net of a tax deduction at the corporate tax rate of 33%, yields unlevered net cash flow from operations of $1916m. These figures are prior to any deduction for replacement or new investment.

3 These figures are prior to any deduction for salvage values. Salvage values are not disclosed, but are presumably embodied within "sales of property, plant and equipment". The latter figures average about 3% of fixed asset purchases. Consequently, salvage values are no more than 3% of the purchase price of the new assets. This is sufficiently small that we treat salvage values as zero.

4 The Financial Statements do project total capital expenditure for the next year of $750m, and this includes some new investment.

are then as shown in Table 1. We also assume that AD_6 is equal to \$760m[5]. Finally, we assume a discount rate of $k = .10$.

Following equations (2), (3) and (4), the current value of the firm is estimated as follows.

$$E(X_6) = E(W_6) - AD_6 = \$2140m - \$760m = \$1380m$$

$$E(V_5) = \frac{\$1380m}{.10 - .02} = \$17,250m$$

$$V_0 = \frac{\$1290m}{1.10} + \frac{\$1330m}{(1.10)^2} + \frac{\$1370m}{(1.10)^3} + \frac{\$1410m}{(1.10)^4} + \frac{\$1450m + \$17,250m}{(1.10)^5} = \$15,875m$$

To express this outcome in terms of the firm's share price on 30.6.2005, we deduct the book value of debt at 30.6.2005 of \$3840m, and divide the residue by the number of ordinary shares at that time of 1,957m, to yield a share value of \$6.15.[6]

We now turn to equation (8). This requires identification of the firm's assets in five years along with their residual depreciation tax savings, current replacement cost (new), residual economic life, full economic life, and the time profile of the depreciation tax savings over the full economic life. Of course, we do not have access to this information but the Financial Statements do offer significant clues in this area. In particular, the firm currently has fixed assets with an aggregate historic cost of \$12,000m, depreciated historic cost of \$4300m, depreciation of \$700m, and it uses the straight-line depreciation methodology. Given the use of straight-line depreciation, the average economic life of the firm's assets is \$12000m/\$700m = 17.1 years. Furthermore, the average residual life of the firm's assets at the present time is \$4300m/\$700m = 6.1years. This reveals that, in five years time, the overwhelming

[5] This is moderately larger than the current figure of \$700m, in recognition of the fact that asset replacements over the next five years will involve replacement costs in excess of the historic costs of the assets that are replaced. The figure of \$760m is actually derived later in implementing equation (8).

[6] Purely in passing, we note that this figure of \$6.15 is within 10% of the share price at that time. Of course, this implies nothing about the validity of the model used to generate the figure because variations in the parameter values used could produce substantial variations in the result.

proportion of the firm's assets will require replacement shortly thereafter, and this is the very situation under which equation (4) will overestimate the value of the firm.

To determine the precise degree of overestimation, it is necessary to make some assumption about the distribution of the economic lives of the firm's assets, subject to the average being 17 years. However, the result is almost insensitive to the particular assumption made. So, we assume that all of the firm's assets have an economic life of 17 years. It follows that all expenditures on replacement investment over the next five years ($650m per year) will involve assets that currently exist and were purchased 12 to 16 years ago. Consistent with inflation in New Zealand consumer prices over the last 15 years, we assume a historic inflation rate of 2% per annum here (Reserve Bank of New Zealand, 2006). The historic cost of each of these five sets of assets must then be $650m discounted at 2% for 17 years, i.e., $464m. The current depreciated historic cost of each set then follows from this historic cost along with the current residual life of the asset. The results for these five sets of assets are shown in the first five rows of Table 2.

Deduction of the aggregate *HC* and *DHC* of the assets to be replaced within the next five years from the figures of $12000m and $4300m reveals that the aggregate *HC* and *DHC* of the remaining assets must be $9680m and $3890m respectively. Since the economic lives of all of these assets is 17 years, then the average residual life of these assets (R) is such that $3890m = $9680m($R$/17). It follows that R is approximately 7 years, and therefore the average period since purchase of the assets must be approximately 10 years. Since there are a range of allocations consistent with the figures of $9680m and $3890m, we arbitrarily specify the cost of the assets purchased 10 years ago as $3000m, with the remainder purchased 9 and 11 years ago. The assets purchased 10 years ago have a depreciated historic cost of $1235m, with the calculation as shown in Table 2. The remaining assets (purchased 9 and 11 years ago) must then have an aggregate *HC* and *DHC* of $6680m and $2655m respectively. Designating the historic costs of the assets purchased 9 and 11 years ago as H_1 and H_3, these values must sum to $6680m and their associated depreciated historic costs of H_1(8/17) and H_3(6/17) must sum to $2655m. The solution is shown in Table 2.

We now turn to the situation in five years. The last three sets of assets in Table 2, along with the five sets of replacements over the next five years, form the complete set of assets in five years, and significant characteristics of them are shown in Table 3. The purchase dates of the assets are shown in the first column. The historic costs (*HC*) of the assets are as described above, and are shown in the second column. The residual lives of the assets in five years (*M*) simply follow from the purchase dates along with the economic life of 17 years, and appear in the third column. The values for AD_6 are simply equal to *HC*/17, and appear in the fourth column. The values for TS_6 in the fifth column are simply 33% of AD_6, i.e., the tax depreciation matches AD_6 and the corporate tax rate is 33%. For each set of assets, the aggregation of these tax savings over the remaining life of the asset, using a discount rate of 10%, is shown in the sixth column. The current replacement costs of the assets new (C_0) reflect the values for *HC* along with inflation of 2% per annum since purchase, as shown in the seventh column of the table. For example, for the assets purchased 11 years ago, at a cost of \$4150m, the current replacement cost (new) is \$4150m compounded at 2% for 11 years, to yield \$5160m.

We now proceed to implement equation (8), starting with $E(Z_6)$ as defined in equation (5) and drawing the relevant data from Tables 1 and 3.

$$E(Z_6) = E(W_6) - TS_6 = \$2140m - \$253m = \$1887m$$

In addition we require the present value of the depreciation tax savings over the life of an asset as a proportion of its cost, which is as follows.

$$\sum_{t=1}^{N} \frac{P_t}{(1+k)^t} = P_t\left[\frac{1}{1.10} + ...\frac{1}{(1.10)^{17}}\right] = .33\left(\frac{1}{17}\right)\left[\frac{1}{1.10} + ...\frac{1}{(1.10)^{17}}\right] = .156$$

Finally, we require the last term in equation (8) for each of the eight sets of assets in Table 3. For the assets purchased 11 years ago, the calculation is as follows.

$$\sum_{t=1}^{M}\frac{TS_{5+t}}{(1.10)^t}-\frac{\frac{C_0(1.02)^{M+5}}{(1.10)^M}\left[1-.156\right]}{1-\frac{(1.02)^{17}}{(1.10)^{17}}}=\$74m-\frac{\frac{\$5160m(1.02)^6}{1.10}(1-.156)}{1-\frac{(1.02)^{17}}{(1.10)^{17}}}=-\$6093m$$

This result along with the results for the other seven sets of assets are shown in the last column of Table 3, and their sum over all eight sets of assets is -\$13,853m. Substitution of these results into equation (8) yields the following.

$$E(V_5)=\frac{\$1887m}{.10-.02}-\$13{,}853m=\$9735m$$

$$V_0=\frac{\$1290m}{1.10}+\frac{\$1330m}{(1.10)^2}+\frac{\$1370m}{(1.10)^3}+\frac{\$1410m}{(1.10)^4}+\frac{\$1450m+\$9735m}{(1.10)^5}=\$11{,}209m$$

So, the estimate from equation (4), of \$15,875m, is 42% larger. To express the result on a per share basis, we again deduct the book value of debt of \$3840m, and divide the residue by the number of ordinary shares of 1,957m, to yield a share value of \$3.76. The estimate from equation (4), of \$6.15, is now 64% larger.

The results here reflect estimates of the current replacement cost of the assets and the expected rate of appreciation; any variation from these estimates will affect the difference in results from equations (4) and (8). So, we examine variations in *historical* inflation rates in asset costs (i_H), which will affect the current replacement cost of assets, and the results are shown in Table 4. As the historical inflation rate increases, the value arising from equation (8) falls and therefore the overestimation resulting from the use of equation (4) becomes even worse. At a historical inflation rate of 4%, the estimate of V_0 from (4) would be 90% too high and the estimate of the share price from (4) would then be 168% too high. All of this suggests that knowledge of the current replacement costs of the firm's assets, along with their economic and residual lives, is very significant information for the purposes of valuing firms. The average values for the last two pieces of information can be deduced from either historic cost or replacement cost valuation of the firm's assets, and average values are sufficient for this purpose. By contrast, the first of these three pieces of information requires replacement cost valuation of the firm's assets. So, Financial Statements based upon replacement cost asset valuations are superior to those based upon historic cost valuations.

5. Conclusion

This paper analyses the issue of the timing of expenditures in replacing fixed assets within the context of valuing firms using the free cash flow approach. Standard practice amongst both practitioners and academics is to assume a smooth pattern in these expenditures past some future point, and such a pattern is improbable. This paper shows how to correctly address this issue, that failure to do so may induce valuation errors in excess of 50%, and presents an example from a company in which this appears to be the case. In conducting the analysis for this company, it is important to know the current replacement costs of the firm's existing assets, their economic lives and their residual lives. The average values for the last two pieces of information can be deduced from either historic cost or replacement cost valuation of the firm's assets, and average values are sufficient for this purpose. By contrast, the first of these three pieces of information requires replacement cost valuation of the firm's assets. So, Financial Statements based upon replacement cost asset valuations are superior to those based upon historic cost valuations.

Table 1: Projected Cash Flows for Telecom New Zealand

	Yr 1	Yr 2	Yr 3	Yr 4	Yr 5	Yr 6
$E(W_t)$	1940	1980	2020	2060	2100	2140
$- R_t$	650	650	650	650	650	
$= E(F_t^u)$	1290	1330	1370	1410	1450	

This table shows expected cash flows from operations for Telecom over the next six years, expected expenditures on replacement investment over the next five years, and expected free cash flows over the next five years (in $m).

Table 2: Characteristics of Telecom's Existing Assets

Purchase Date	*HC*	*DHC*
-16	\$464	\$464(1/17) = \$27
-15	\$464	\$464(2/17) = \$55
-14	\$464	\$464(3/17) = \$82
-13	\$464	\$464(4/17) = \$109
-12	\$464	\$464(5/17) = \$137
-11	\$4150	\$4150(6/17) = \$1464
-10	\$3000	\$3000(7/17) = \$1235
-9	\$2530	\$2530(8/17) = \$1191
	$\sum$ = \$12000	$\sum$ = \$4300

This table shows a possible distribution for the purchase dates, historic costs and depreciated historic costs for Telecom's existing assets. This distribution is consistent with the aggregate historic cost of the firm's existing assets, their depreciated historic cost, the current depreciation and the firm's use of straight-line depreciation.

Table 3: Characteristics of Telecom's Assets in Five years Time

Pchd	*HC*	*M*	AD_6	TS_6	$\sum_{t=1}^{M} \frac{TS_{5+t}}{(1.10)^t}$	C_0	[ ]
-11	$4150	1	$244	$81	$74	$5160	-$6093
-10	$3000	2	$176	$58	$101	$3657	-$3952
-9	$2530	3	$149	$49	$122	$3024	-$2985
1	$650	13	$38	$13	$92	$637	-$248
2	$650	14	$38	$13	$96	$625	-$184
3	$650	15	$38	$13	$99	$613	-$156
4	$650	16	$38	$13	$102	$601	-$129
5	$650	17	$38	$13	$104	$589	-$106
			$\sum = \$760$	$\sum = \$253$			$\sum = -\$13853$

This table shows characteristics of Telecom's assets in five years, arising from the distribution in Table 2 and the expenditures on asset replacements over the next five years.

Table 4: The Effect of Variations in Historical Inflation Rates

Model	Firm Value	Share Value
Equation (4)	$15.875b	$6.15
Equation (8), $i_H = 0$	$13.726b	$5.05
Equation (8), $i_H = .02$	$11.209b	$3.76
Equation (8), $i_H = .04$	$8.315b	$2.29

This table shows the effect of historical inflation rates in fixed asset prices upon the estimates of Telecom's value using equation (8). Results for equation (4) are not dependent upon such historical inflation rates. Comparison between the results for (8) and (4) reveals the degree of error arising from the latter.

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