

Modelling Exchange-Traded Barrier Options traded in the Australian Options Market

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Abstract

Barrier options traded in the Australian market vary considerably in terms of the extent to which the barrier is monitored. They also vary in terms of the location of the barrier level relative to the exercise price. This paper examines the impact of these differences on prices and also on deltas and gammas. Values of these hedge parameters are of interest as, prima facie, they may explain earlier findings that observed barrier option prices are higher than expected. Contrary to the finding of Chance (1994), we find that it is not possible to generalise results concerning hedge parameter values to all barrier options. We find that options examined by Easton et al (2004) do not display instability of deltas near barrier levels, while those examined by Chance and those recently issued in Australia do display delta instability. As the options examined by Easton et al do not display delta instability, their apparent overpricing cannot be attributed to hedging difficulties.

Key Words: Barrier Options, Barrier Monitoring, Hedging

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1. Introduction

Barrier options are path-dependent options for which the existence of the option depends on whether the underlying asset price has breached a barrier level during the options life. Such options have been actively traded in over-the-counter markets since the late 1980s. In addition, an exchange-traded barrier option was introduced in 1991 on the Chicago Board Options Exchange and the American Exchange.

Such exchange-traded barrier options are now commonplace in the Australian options market. Barrier options were issued by Bankers Trust in 1998 and 1999. In late 2003 and early 2004, further barrier options were introduced by Citigroup, Deutsche Bank, Macquarie Bank and UBS Investment Bank. The success of these products has been such that by as early as February 2004, their trading volume exceeded that of traditional warrants.

Closed-form analytical solutions for valuing simple barrier options have been provided by Merton (1973), Cox and Rubinstein (1985) and Rubinstein and Reiner (1991). Closed-form analytical solutions to more complex barrier options have been provided by Chesney, Jeanblanc-Picque and Yor (1997), Haber, Schonbucher and Wilmott (1999) and Hugonnier (1999). However, these models all assume that the barrier is monitored

continuously. Numerous studies have highlighted inaccuracies that result from applying such analytical solutions when the barrier is monitored discretely.¹

The options traded in the Australian market vary considerably in terms of the extent to which the barrier is monitored. They also vary in terms of the location of the barrier level relative to the exercise price. The options issued by Bankers Trust in 1998 and 1999 were on the All Ordinaries Price Index and required that the closing level of the index was at or above the barrier level for three consecutive days during the life of the option. Barrier options such as these, where the barrier is triggered only after the price of the underlying asset has breached the barrier for a prescribed period of time, are termed Parisian options. Of the options currently traded, those issued by Deutsche Bank, Macquarie Bank and UBS Investment Bank all assume continuous monitoring of the barrier. Some of the Citigroup barrier options also assume continuous monitoring, while for other Citigroup barrier options the option continues to exist provided the closing share price does not breach the barrier. Those options with continuous monitoring have the barrier level set equal to the exercise price, while the Citigroup options with monitoring at the close have barrier levels that exceed (are less than) the exercise price for call (put) options.

This paper examines the impact of these differences on prices and also on deltas and gammas. Values of these hedge parameters are of interest as they may explain the finding of Easton et al (2004) that observed barrier option prices are higher than expected.

Following Flesaker (1992) and Chance (1994), Easton et al suggest their finding may be

¹ See for example studies by Hudson (1992), Chance (1994), Derman et al (1995), Kat and Verdonk (1995), Anderson and Brotherton-Ratcliffe (1996), Berger (1996), Chesney et al (1997), Levy and Manton (1997) and Ahn et al (1999).

due to barrier options having very high gammas near the barrier level. This indicates instability of deltas at these critical barrier levels; an instability that Chance notes makes hedging such options virtually impossible. As a result, writers of barrier options may require a premium not included in standard pricing models.

Monte Carlo simulation is used to value the barrier options and to generate the hedge parameter values. Monte Carlo simulation is used rather than lattice-based approaches as suggested by Cox and Rubinstein (1985, pp. 409-412), Hudson (1992), Boyle and Lau (1994), Derman et al (1995), Kat and Verdonk (1995), Ritchken (1995), Cheuk and Vorst (1996), Avellaneda and Wu (1999) and Vetzal and Forsyth (1999). As noted by Ahn (1999), these models also yield inaccurate values when the barrier is monitored discretely.

Section 2 describes the barrier options traded on the Australian Stock Exchange. Section 3 examines the valuation of each of these options under alternative assumptions with respect to the monitoring of the barrier. Section 4 presents a summary of the results.

2. Barrier Options traded on the Australian Stock Exchange

Easton et al (2004) examined eight different barrier listed on the Australian Stock Exchange over the period August 1998 to September 1999. The common feature of all eight options was that the barrier was monitored discretely at the daily closing value of the underlying All Ordinaries Price Index and that they were knock-out barrier options. Further, for the call (put) options the barrier level was below (exceeded) the exercise

price. Three of these options were Parisian, requiring that the closing level of the index breached the barrier level for three consecutive days during the life of the option.

The options issued by Citigroup, Deutsche Bank, Macquarie Bank and UBS Investment Bank are offered across a range of underlying shares and indices. They are all issued deep-in-the-money and in the case of those issued by Deutsche Bank, Macquarie Bank and UBS Investment Bank the exercise price and barrier level are the same. All barrier options issued by these banks provide for continuous monitoring of the barrier level.

Options issued by Citigroup are of two types. One type is of the same form as those listed by the other issuers, with continuous monitoring of the barrier level and with the exercise price equal to the barrier level.

The other type of option listed by Citigroup provides for discrete monitoring at the daily closing value of the underlying share price, with the barrier level set above the exercise price for call options and below the exercise price for put options.

This type of option also provides a payout when the barrier is breached. The payout is equal to the maximum of zero and the difference between the exercise price and the average share price for all trades over the five trading days following the day on which the barrier level is breached. This payout is therefore that of an embedded Asian option.²

² For details of Asian options, see Kemna and Vorst (1990), Turnbull and Wakeman (1991) and German and Yor (1993).

3. Results

To ensure accurate valuation, the term to maturity of the option was divided into 500 x n discrete periods, where n is the number of days to maturity. To ensure low standard errors, the Monte Carlo simulation relied on a total of 100 000 iterations. The antithetic variance reduction technique was used to further reduce the variance of the simulation results. In addition, the control variate technique was employed, using Merton's (1973) barrier option pricing model as the control variable.

Table 1 presents the results for a call option examined by Easton et al (2004). This option had a barrier level of 2350 points and an exercise level of 2800 points.³ If the closing value of the index was at or below 2350 points *for three consecutive days during the life of the option*, the option terminated worthless. Providing that the barrier had not been breached, on contract maturity the holder received \$0.005 for each point the index was above 2800 points. In undertaking these simulations, the term to maturity was set at 90 days, the standard deviation of the underlying returns was 20 per cent per annum, the risk free interest rate was 5 per cent per annum, and the continuous dividend yield was 4 per cent per annum.

Columns 2 to 4 report the option values and values of the hedge parameters under continuous monitoring of the barrier. Columns 5 to 7 report the option values and hedging parameters under daily monitoring of the barrier. Columns 8 to 10 report the

³ The hedging characteristics of this option will be similar to those for other call options where the barrier level is below the exercise level.

option values and values of the hedging parameters where the barrier is breached if the closing value of the index is at or below the barrier level for three consecutive days.

The results show that while option prices differ due to different specifications as to the monitoring of the barrier, for this option the values of the hedging parameters are not substantially affected by these specifications. While the delta of the option with continuous monitoring falls near the barrier level, of importance for hedging is that the gammas are uniformly small. As the options issued by Bankers Trust had either daily or Parisian monitoring of the barrier, where the deltas are more uniform than under continuous monitoring, hedging difficulties would appear not to be an explanation for their apparent overpricing.

The results for a put option examined by Easton et al (2004) are presented in Table 2. This option had a barrier level of 2825 points and an exercise level of 2550 points.⁴ If the closing value of the index was at or above 2825 points *for three consecutive days during the life of the option*, the option terminated worthless. Providing that the barrier had not been breached, on contract maturity the holder received \$0.005 for each point the index was above 2550 points. In terms of the affect of different specifications as to the monitoring of the barrier, the results are very similar to those reported for the call option with the gammas being uniformly small.

⁴ The hedging characteristics of this option will be similar to those for other put options where the barrier level is above the exercise level.

These results are at odds with those reported by Chance (1994) and replicated in Table 3. The call option examined by Chance had an exercise price of \$300 and a barrier level of \$330. If the barrier level was breached, the option was automatically exercised with a payoff of \$30. The term to maturity was 60 days, the standard deviation of the underlying returns was 20 per cent per annum, and the risk free interest rate was 7 per cent per annum.⁵

Under the assumption of continuous monitoring, the delta rises to a peak at a price level around \$320 to \$325, and falls dramatically from 0.64 at \$329 to zero at \$330. Under daily monitoring, the delta falls sharply but less dramatically from 0.53 at \$329 to 0.44 at \$330. For the Parisian option the fall is even less, falling from 0.53 at \$329 to 0.50 at \$330. The instability of deltas suggests that hedging near the barrier level may be particularly difficult for options where the barrier is monitored continuously but less so where the barrier is monitored daily, and less so again for Parisian options. This difference in turn suggests that apparent overpricing of barrier options derived from pricing models that ignore impediments to hedging is likely to be greatest for those where the barrier is monitored continuously, and less for Parisian options.

The put option examined by Chance had an exercise price of \$330 and a barrier level of \$300. Again, if the barrier level was breached, the option was automatically exercised with a payoff of \$30. The term to maturity was 60 days, the standard deviation of the underlying returns was 20 per cent per annum, and the risk free interest rate was 7 per cent per annum.

⁵ Your results are very similar to those of Chance, although the increased stability of our results may be noted.

As reported in Panel B of Table 3, under continuous monitoring, deltas again show instability near the barrier, with delta increasing from -0.929 at $\$301$ to zero at $\$300$. Again, instability of deltas is far less of a problem under the alternative monitoring regimes.

Chance (1994, p. 579) argued that the results replicated in Table 3 (Table 5 in his paper) would be generalisable to all types of barrier options. An examination of the results in Tables 1 and 2 demonstrates that this is not the case. Those results demonstrate that hedging difficulties are not present where for call (put) options the barrier level is below (exceeds) the exercise price.

However, hedging difficulties are present for the currently traded options where the exercise price is set equal to the barrier level. For these options the exercise price and the barrier level were set at $\$1$, the term to maturity was set at 90 days, the standard deviation of the underlying returns was 30 per cent per annum, the risk free interest rate was 5 per cent per annum, and the dividend yield was assumed to be zero.

As shown in Table 4, while these call options are designed to have a delta of one it may be noted that close to the barrier delta exceeds one. Instability of delta, especially for the options where the barrier is continuously monitored is extreme, with delta falling from 1.150 at $\$1.01$ to zero at $\$1.00$. Similarly, the put options are designed to have a delta of

negative one but the delta is greater than negative one for underlying asset values close to the barrier. Again, instability of delta is extreme, with the delta for the options where the barrier is monitored continuously increasing from -0.868 at $\$0.99$ to zero at $\$1.00$. Difficulties in implementing hedging strategies are likely to be particularly great for these options.

Table 5 examines options of the type issued by Citigroup. Barrier levels for the call options were set at $\$1.02$ and $\$1.15$ to reflect the range of barrier levels set in the options issued by Citigroup. Similarly, barrier levels for the put options were set at $\$0.85$ and $\$0.98$.⁶ As noted above, these options have a payout equal to the maximum of zero and the difference between the exercise price and the average share price for all trades over the five trading days following the day on which the barrier level is breached. This payout is therefore that of an embedded Asian option. The embedded Asian options reduce the significance of the closing price and reduce the effects of any abnormal movements in the underlying asset. In doing so they ameliorate any price distortions that may arise because of a lack of depth in the market of the underlying asset.

It may be noted that the presence of the embedded Asian option also reduces the instability of the deltas around barrier levels. To that extent these options do not present the same hedging difficulties as those reported in Table 4. However, the evidence is mixed, with very high gammas present at the barrier levels.

⁶ An examination of options issued by Citigroup showed that for call options the barrier levels ranged from 2 to 15 per cent above the exercise price, while for put options the barrier levels ranged from 2 to 15 per cent below the exercise price.

4. Summary

This paper examined the impact of differences in the extent to which the barrier level in barrier options is monitored. The impact of the location of the barrier level relative to the exercise price was also examined. Differences were examined in prices and also in deltas and gammas. Contrary to the finding of Chance (1994), we found that it is not possible to generalise results concerning hedge parameter values to all barrier options. The options examined by Easton et al (2004) do not display instability of deltas near barrier levels, while those examined by Chance and those recently issued in Australia do display delta instability. As the options examined by Easton et al do not display delta instability, their apparent overpricing cannot be attributed to hedging difficulties.

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Table 1**Call Option Issued by Bankers Trust**

Index	Value of Value Option \$	Delta	Gamma	Value of Option \$	Delta	Gamma	Value of Option \$	Delta	Gamma
2050	0	0	0	0	0	0	0	0	0
2150	0	0	0	0	0	0	0	0	0
2250	0	0	0	0	0	0	0	0	0.001
2275	0	0	0	0	0	0	0.001	0	0.001
2300	0	0	0	0	0.001	0.001	0.002	0.023	0.001
2325	0	0	0	0.002	0.028	0.001	0.006	0.043	0.001
2335	0	0	0	0.003	0.041	0.001	0.009	0.054	0.001
2345	0	0	0	0.005	0.053	0.001	0.011	0.056	0.001
2346	0	0	0	0.006	0.054	0.001	0.011	0.057	0.001
2347	0	0	0	0.006	0.055	0.001	0.012	0.057	0.001
2348	0	0	0	0.006	0.056	0.001	0.012	0.058	0.001
2349	0	0	0	0.007	0.057	0.001	0.012	0.058	0.001
2350	0	0	0	0.007	0.058	0.001	0.012	0.059	0.001
2351	0.001	0.086	0	0.007	0.059	0.001	0.013	0.060	0.001
2352	0.001	0.086	0	0.008	0.060	0.001	0.013	0.060	0.001
2353	0.002	0.087	0	0.008	0.061	0.001	0.014	0.061	0.001
2354	0.002	0.087	0	0.008	0.061	0.001	0.014	0.062	0.001
2355	0.002	0.088	0	0.009	0.062	0.001	0.014	0.063	0.001
2365	0.007	0.089	0	0.012	0.069	0.001	0.017	0.067	0.001
2375	0.012	0.091	0	0.016	0.080	0.001	0.021	0.072	0.001
2400	0.024	0.097	0	0.027	0.092	0	0.031	0.082	0
2425	0.036	0.103	0	0.038	0.097	0	0.041	0.089	0
2450	0.050	0.116	0	0.052	0.108	0	0.054	0.100	0

Columns 2 to 4 report the option values and values of the hedging parameters under continuous monitoring of the barrier. Columns 5 to 7 report the option values and hedging parameters under daily monitoring of the barrier. Columns 8 to 10 report the option values and values of the hedging parameters where the barrier is breached if the closing value of the index is at or below the barrier level for three consecutive days. The exercise level is 2800 and the barrier level is 2350. Providing the barrier has not been breached, on contract maturity the holder receives \$0.005 for each point the index is above 2800. If the barrier level is breached, the option terminates worthless. The term to maturity is 90 days, the standard deviation of the underlying returns is 20 per cent per annum, the risk free rate of interest is 5 per cent per annum, and the continuous dividend yield is 4 per cent per annum.

Table 2**Put Option Issued by Bankers Trust**

Index	Value of			Value of			Value of		
Value	Option \$	Delta	Gamma	Option \$	Delta	Gamma	Option \$	Delta	Gamma
2725	0.141	-0.300	0.001	0.152	-0.272	0.001	0.166	-0.261	0.001
2750	0.106	-0.292	0	0.119	-0.277	0.001	0.135	-0.248	0.001
2775	0.070	-0.283	0	0.085	-0.265	0.001	0.105	-0.225	0.001
2800	0.035	-0.275	0	0.053	-0.235	0.002	0.076	-0.193	0.002
2810	0.022	-0.271	0	0.041	-0.219	0.002	0.065	-0.178	0.002
2820	0.008	-0.268	0	0.031	-0.200	0.002	0.055	-0.161	0.002
2821	0.006	-0.268	0	0.030	-0.198	0.002	0.053	-0.159	0.002
2822	0.005	-0.267	0	0.029	-0.196	0.002	0.053	-0.157	0.002
2823	0.004	-0.267	0	0.028	-0.194	0.002	0.052	-0.155	0.002
2824	0.002	-0.266	0	0.027	-0.191	0.002	0.051	-0.154	0.002
2825	0	0	0	0.026	-0.189	0.002	0.050	-0.152	0.002
2826	0	0	0	0.025	-0.187	0.002	0.049	-0.150	0.002
2827	0	0	0	0.024	-0.185	0.002	0.048	-0.148	0.002
2828	0	0	0	0.023	-0.183	0.002	0.047	-0.146	0.002
2829	0	0	0	0.022	-0.180	0.002	0.046	-0.144	0.002
2830	0	0	0	0.021	-0.179	0.002	0.044	-0.140	0.002
2840	0	0	0	0.014	-0.135	0.008	0.036	-0.122	0.002
2850	0	0	0	0.009	-0.088	0.003	0.028	-0.108	0.001
2875	0	0	0	0.002	-0.028	0.002	0.014	-0.075	0.001
2900	0	0	0	0	-0.001	0	0.006	-0.050	0.001
2925	0	0	0	0	0	0	0.002	-0.031	0.001
3025	0	0	0	0	0	0	0.000	-0.018	0
3125	0	0	0	0	0	0	0	0	0

Columns 2 to 4 report the option values and values of the hedging parameters under continuous monitoring of the barrier. Columns 5 to 7 report the option values and hedging parameters under daily monitoring of the barrier. Columns 8 to 10 report the option values and values of the hedging parameters where the barrier is breached if the closing value of the index is at or above the barrier level for three consecutive days. The exercise level is 2550 and the barrier level is 2825. Providing the barrier has not been breached, on contract maturity the holder receives \$0.005 for each point the index is below 2550. If the barrier level is breached, the option terminates worthless. The term to maturity is 90 days, the standard deviation of the underlying returns is 20 per cent per annum, the risk free rate of interest is 5 per cent per annum, and the continuous dividend yield is 4 per cent per annum.

Table 3 – Panel A**Barrier Call Option Examined by Chance (1994)**

Index	Value of			Value of			Value of		
	Value	Option \$	Delta	Gamma	Option \$	Delta	Gamma	Option \$	Delta
290	6.41	0.396	0.020	6.23	0.382	0.018	6.05	0.364	0.009
300	11.01	0.527	0.014	10.63	0.512	0.015	10.15	0.486	0.009
310	16.82	0.632	0.007	16.17	0.619	0.007	15.30	0.550	0.009
320	23.32	0.671	-0.000	22.38	0.649	-0.001	21.03	0.611	0.009
325	26.64	0.662	-0.004	25.56	0.630	-0.021	23.99	0.600	-0.009
326	27.32	0.658	-0.004	26.21	0.611	-0.025	24.56	0.587	-0.012
327	28.00	0.654	-0.005	26.85	0.581	-0.029	25.16	0.572	-0.016
328	28.63	0.648	-0.006	27.41	0.550	-0.033	25.70	0.556	-0.019
329	29.29	0.643	-0.006	28.00	0.527	-0.047	26.26	0.529	-0.023
330	30.00	0	0	28.50	0.442	-0.071	26.82	0.499	-0.027
331	30.00	0	0	28.90	0.363	-0.073	27.29	0.469	-0.030
332	30.00	0	0	29.25	0.298	-0.070	27.78	0.439	-0.034
333	30.00	0	0	29.52	0.219	-0.065	28.24	0.396	-0.037
334	30.00	0	0	29.70	0.151	-0.055	28.57	0.351	-0.041
335	30.00	0	0	29.82	0.102	-0.035	28.88	0.309	-0.044
340	30.00	0	0	29.99	0.004	-0.004	29.75	0.082	-0.025

Columns 2 to 4 report the option values and values of the hedging parameters under continuous monitoring of the barrier. Columns 5 to 7 report the option values and hedging parameters under daily monitoring of the barrier. Columns 8 to 10 report the option values and values of the hedging parameters where the barrier is breached if the closing value of the index is at or above the barrier level for three consecutive days. The exercise price is \$300 and the barrier level is \$330. If the barrier level is breached, the option is automatically exercised with a payoff of \$30. The term to maturity is 60 days, the standard deviation of the underlying returns is 20 per cent per annum, and the risk free rate of interest is 7 per cent per annum.

Table 3 – Panel B**Barrier Put Option Examined by Chance (1994)**

Index Value	Value of Option \$	Delta	Gamma	Value of Option \$	Delta	Gamma	Value of Option \$	Delta	Gamma
290	30.00	0	0	29.99	-0.003	-0.002	29.78	-0.069	-0.016
295	30.00	0	0	29.83	-0.131	-0.075	28.74	-0.306	-0.087
296	30.00	0	0	29.69	-0.208	-0.097	28.32	-0.386	-0.110
297	30.00	0	0	29.44	-0.321	-0.112	27.81	-0.429	-0.086
298	30.00	0	0	29.10	-0.424	-0.115	27.26	-0.482	-0.060
299	30.00	0	0	28.63	-0.512	-0.112	26.60	-0.529	-0.044
300	30.00	0	0	28.04	-0.597	-0.082	25.91	-0.570	-0.039
301	29.00	-0.929	-0.004	27.36	-0.676	-0.073	25.22	-0.606	-0.033
302	28.09	-0.913	0.014	26.58	-0.752	-0.055	24.46	-0.636	-0.027
303	27.20	-0.897	0.016	25.77	-0.810	-0.031	23.73	-0.661	-0.022
304	26.34	-0.881	0.016	24.97	-0.838	-0.013	22.99	-0.679	-0.016
305	25.42	-0.865	0.016	24.14	-0.835	0.018	22.28	-0.692	-0.010
310	21.36	-0.785	0.016	20.30	-0.748	0.017	18.82	-0.679	0.016
320	14.37	-0.621	0.017	13.74	-0.582	0.016	12.91	-0.530	0.014
330	9.08	-0.454	0.017	8.73	-0.427	0.015	8.29	-0.394	0.013
340	5.36	-0.286	0.017	5.23	-0.289	0.014	5.07	-0.273	0.012

Columns 2 to 4 report the option values and values of the hedging parameters under continuous monitoring of the barrier. Columns 5 to 7 report the option values and hedging parameters under daily monitoring of the barrier. Columns 8 to 10 report the option values and values of the hedging parameters where the barrier is breached if the closing value of the index is at or above the barrier level for three consecutive days. The exercise price is \$330 and the barrier level is \$300. If the barrier level is breached, the option is automatically exercised with a payoff of \$30. The term to maturity is 60 days, the standard deviation of the underlying returns is 20 per cent per annum, and the risk free rate of interest is 7 per cent per annum.

Table 4 – Panel A**Call Options Issued By Citigroup, Deutsche Bank, Macquarie Bank and UBS Investment Bank**

Asset Price \$	Value of Option \$	Delta	Gamma	Value of Option \$	Delta	Gamma	Value of Option \$	Delta	Gamma
0.75	0	0	0	0	0	0	0	0	0
0.85	0	0	0	0	0	0	0	0	0
0.90	0	0	0	0	0	0	0	0	0
0.95	0	0	0	0	0.001	0	0.001	0.159	0.048
0.96	0	0	0	0	0.004	0.004	0.002	0.236	0.067
0.97	0	0	0	0	0.116	0.068	0.005	0.390	0.087
0.98	0	0	0	0.002	0.222	0.112	0.010	0.562	0.085
0.99	0	0	0	0.005	0.487	0.150	0.016	0.713	0.068
1.00	0	0	0	0.012	0.806	0.225	0.025	0.840	0.048
1.01	0.012	1.150	-0.002	0.021	1.029	-0.002	0.034	0.923	0.028
1.02	0.023	1.139	-0.002	0.031	1.025	-0.002	0.043	0.954	0.009
1.03	0.034	1.120	-0.002	0.042	1.022	-0.002	0.053	0.956	-0.008
1.04	0.045	1.095	-0.001	0.052	1.018	-0.002	0.062	0.954	-0.007
1.05	0.056	1.061	-0.001	0.062	1.016	-0.001	0.072	0.953	-0.006
1.10	0.109	1.047	-0.001	0.113	1.005	-0.001	0.120	0.951	0.001
1.15	0.161	1.031	-0.001	0.164	1.000	-0.001	0.167	0.956	0.001
1.25	0.261	1.007	0.001	0.263	1.000	0.001	0.264	0.988	0.002

Columns 2 to 4 report the option values and values of the hedging parameters under continuous monitoring of the barrier. Columns 5 to 7 report the option values and hedging parameters under daily monitoring of the barrier. Columns 8 to 10 report the option values and values of the hedging parameters where the barrier is breached if the closing value of the index is at or below the barrier level for three consecutive days. The exercise price and the barrier level are both \$1.00. If the barrier level is breached, the option terminates worthless. The term to maturity is 90 days, the standard deviation of the underlying returns is 30 per cent per annum, the risk free rate of interest is 5 per cent per annum, and the continuous dividend yield is assumed to be zero.

Table 4 – Panel B**Put Options Issued By Citigroup, Deutsche Bank, Macquarie Bank and UBS Investment Bank**

Asset Price\$	Value of			Value of			Value of		
	Option \$	Delta	Gamma	Option \$	Delta	Gamma	Option \$	Delta	Gamma
0.75	0.238	-0.988	0.001	0.239	-0.985	0.001	0.239	-0.958	0
0.85	0.140	-0.978	0.002	0.142	-0.953	0.003	0.144	-0.915	0.009
0.90	0.092	-0.952	0.003	0.095	-0.919	0.004	0.100	-0.868	0.010
0.95	0.045	-0.911	0.005	0.050	-0.870	0.006	0.058	-0.806	0.012
0.96	0.036	-0.901	0.005	0.042	-0.862	0.008	0.050	-0.780	0.014
0.97	0.027	-0.891	0.005	0.033	-0.859	0.012	0.042	-0.750	0.016
0.98	0.018	-0.880	0.006	0.025	-0.850	0.021	0.034	-0.711	0.028
0.99	0.009	-0.868	0.006	0.016	-0.762	0.062	0.027	-0.633	0.050
1.00	0	0	0	0.009	-0.581	0.113	0.019	-0.513	0.071
1.01	0	0	0	0.004	-0.372	0.096	0.013	-0.382	0.059
1.02	0	0	0	0.002	-0.197	0.079	0.008	-0.276	0.047
1.03	0	0	0	0	-0.089	0.061	0.005	-0.195	0.035
1.04	0	0	0	0	-0.031	0.039	0.002	-0.122	0.021
1.05	0	0	0	0	-0.002	0.004	0.001	-0.090	0.010
1.10	0	0	0	0	0	0	0	-0.005	0.002
1.15	0	0	0	0	0	0	0	0	0
1.25	0	0	0	0	0	0	0	0	0

Columns 2 to 4 report the option values and values of the hedging parameters under continuous monitoring of the barrier. Columns 5 to 7 report the option values and hedging parameters under daily monitoring of the barrier. Columns 8 to 10 report the option values and values of the hedging parameters where the barrier is breached if the closing value of the index is at or above the barrier level for three consecutive days. The exercise price and the barrier level are both \$1.00. If the barrier level is breached, the option terminates worthless. The term to maturity is 90 days, the standard deviation of the underlying returns is 30 per cent per annum, the risk free rate of interest is 5 per cent per annum, and the continuous dividend yield is assumed to be zero.

Table 5 – Panel A

Call Options Issued By Citigroup

Barrier = \$1.02

Asset Price \$	Value of Option \$	Delta	Gamma	Value of Option \$	Delta	Gamma	Value of Option \$	Delta	Gamma
0.75	0	0	0	0	0	0	0	0	0
0.85	0	0	0	0	0	0	0	0	0
0.90	0	0	0	0	0.006	0	0	0.016	0.007
0.95	0	0.021	0.009	0	0.033	0.014	0.001	0.050	0.029
0.96	0	0.054	0.022	0.001	0.075	0.029	0.003	0.117	0.046
0.97	0.001	0.121	0.037	0.002	0.150	0.045	0.005	0.238	0.063
0.98	0.003	0.251	0.630	0.004	0.272	0.061	0.008	0.352	0.068
0.99	0.005	0.388	0.079	0.007	0.414	0.077	0.012	0.510	0.073
1.00	0.010	0.550	0.082	0.012	0.564	0.093	0.018	0.662	0.078
1.01	0.015	0.718	0.085	0.019	0.766	0.109	0.026	0.824	0.084
1.02	0.023	1.077	0.800	0.028	1.053	0.350	0.035	1.000	0.120
1.03	0.034	1.074	-0.002	0.038	1.050	-0.001	0.046	1.000	-0.001
1.04	0.045	1.070	-0.002	0.048	1.047	-0.001	0.055	1.000	-0.001
1.05	0.056	1.067	-0.002	0.059	1.045	-0.001	0.066	0.999	-0.001
1.10	0.109	1.049	-0.002	0.111	1.031	-0.001	0.115	0.995	-0.001
1.15	0.161	1.031	-0.001	0.162	1.018	-0.001	0.165	0.990	-0.001
1.25	0.262	0.995	-0.002	0.263	0.992	-0.001	0.264	0.980	-0.001

Barrier = \$1.15

Asset Price \$	Value of Option \$	Delta	Gamma	Value of Option \$	Delta	Gamma	Value of Option \$	Delta	Gamma
0.75	0	0	0	0	0	0	0	0	0
0.85	0	0	0	0	0	0	0	0	0
0.90	0	0	0	0	0.002	0	0	0.003	0.001
0.95	0	0.003	0.001	0	0.035	0.001	0.001	0.007	0.001
0.96	0	0.034	0.013	0.001	0.076	0.011	0.002	0.145	0.026
0.97	0.001	0.076	0.040	0.002	0.127	0.038	0.004	0.207	0.037
0.98	0.003	0.181	0.068	0.004	0.228	0.063	0.006	0.292	0.048
0.99	0.005	0.339	0.083	0.007	0.374	0.077	0.010	0.395	0.055
1.00	0.009	0.510	0.086	0.012	0.528	0.075	0.015	0.508	0.057
1.01	0.015	0.674	0.076	0.017	0.665	0.061	0.020	0.617	0.052
1.02	0.023	0.807	0.055	0.024	0.774	0.048	0.027	0.713	0.044
1.03	0.032	0.898	0.037	0.033	0.857	0.036	0.035	0.793	0.037
1.04	0.041	0.955	0.021	0.042	0.918	0.025	0.043	0.860	0.030
1.05	0.051	0.986	0.010	0.051	0.960	0.017	0.052	0.913	0.024
1.10	0.100	1.000	0	0.101	0.998	-0.002	0.101	1.000	0
1.15	0.150	1.093	0.400	0.152	1.301	1.0000	0.154	1.000	0
1.25	0.258	1.041	-0.003	0.259	1.037	0	0.260	1.000	0

Columns 2 to 4 report the option values and values of the hedging parameters under continuous monitoring of the barrier. Columns 5 to 7 report the option values and hedging parameters under daily monitoring of the barrier. Columns 8 to 10 report the option values and values of the hedging parameters where the barrier is breached if the closing value of the index is at or below the barrier level for three consecutive days. The exercise price and the barrier level are both \$1.00. If the barrier level is breached, the option terminates worthless. The term to maturity is 90 days, the standard deviation of the underlying returns is 30 per cent per annum, the risk free rate of interest is 5 per cent per annum, and the continuous dividend yield is assumed to be zero.